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THE HISTORY AND DEVELOPMENT OF NONLINEAR STELLAR PULSATION CODES

by

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ABSTRACT

This review is limited to the history and development of nonlinear stellar pulsation codes and methods. Starting with the digital computer and the method of pseudo-viscosity, that made it feasible to solve the equations of hydrodynamics coupled with heat flow, till the present time with our super computers and new techniques of hydrodynamics the discussion proceeds. The narrative includes examples of practical interest in the application of these numerical methods to problems in stellar pulsation such as Cepheid mass discrepancy, the delineation of the RR Lyrae instability strip, and the question of the development of double-mode pulsation as observed in Cepheids, RR Lyrae and other variable stars.

I. INTRODUCTION

The history of nonlinear pulsation codes really begins with the development of the computer. Previous to this time, Baker and others had established one-zone models of pulsation but it wasn't until the 50s when the digital computers became available that a solution to a set of nonlinear difference equations of coupled hydrodynamics and radiation flow were possible. Previous to this time Eddington, Zhevakin, Cox and Whitney, and others had determined that the de-stabilization in stars, that caused the observed pulsations, was due to the ionization of hydrogen and helium in the atmospheres of such variables as Cepheids, R. R. Lyrae and W Virginis. The finite difference equations of nonlinear hydrodynamics were first used on the CPC, the Eniac and the Maniac at Los Alamos. This work was the genesis of the early pulsation codes of Christy and Cox, et al.. The first nonlinear pulsation calculations were probably done on the IBM 704. We will discuss these early developments as well as the more recent improvements in hydro and radiative transfer that have made important contributions to our understanding of the properties of stellar pulsation.

In Section II, we discuss the pioneering work of Christy and Cox as they battled with machine language coding and the IBM 704 comp. rs. In Section III, we discuss the improvements to solve the equations of multigroup transfer coupled with hydrodynamics and the development of a relaxation scheme to study limiting amplitude and improvements in the hydrodynamics (non-Lagrangian). In Section IV, we discuss the present status of nonlinear stellar pulsation and its application to details of light curves, "resonance" and the vexing problem of double-mode pulsation. Finally in Section V, we detail some possible improvements to present codes and the need for time dependent convection and the possibility of two-dimensional models.

References will be limited to the Los Alamos, Goddard Conferences (LG), to the original publication of the various codes and a review article by J. Cox (1974).

II. EARLY DEVELOPMENTS

With the advent of the computer and the establishment of methods to solve the equations of hydrodynamics (pseudo viscosity) the first nonlinear pulsation calculations were made. Some of the pioneers were Christy (1964) at Cal Tech and A. Cox, et al. (1969) at Los Alamos. Their codes were based on the early work of Richtmyer and Von Neuman and the development of the method of pseudo viscosity, which made it possible to automatically treat the growth of compression waves into shocks in the nonlinear equations while maintaining stability. Christy studied questions of mode transition, limiting amplitude and the Hertzsprung sequence for mass determination. Developing a series of RR Lyrae models he determined the transition from the first overtone at the blue edge of the instability strip through an either/or condition to the fundamental pulsators at the red edge of a strip. The idea of a transition line from the 1st overtone to fundamental in period was confirmed by some more recent calculations of Stellingwerf. These lines are sensitive to pseudo viscosity as well as the addition of radiation pressure in the equations Christy at first did not include the radiation pressure in his equations, which shifted his transition line somewhat (LG-1). Another of Christy's studies addressed the question of the "bump" sequence as observed in Cepheid variables (Hertzsprung). Using a series of Cepheid models he determined that the phase of the observed "bumps" depended on luminosity in the Hertzprung-Russell diagram in the following way,

$$\pi_{tr} = 0.057 \; L \; 0.6 \text{ (days)} \quad .$$

where L is the Luminosity in solar units.

Following Christy's work Stobbie (1969) completed a detailed study where he considered the effects of zoning, pseudo-viscosity and the weighting of the opacities on the bump phase, his results are generally in support of Christy. A more recent report by Fadeyev (1981) supports these earlier results, which means that the question of the Cepheid "bump" masses still remains.

The Cox's approached the problem of nonlinear pulsation from the idea of understanding the mechanisms of pulsation rather than to study the modes of pulsation in stars. In the Cox's study, they included only a shallow envelope (10% of the mass) in their models and limited the excitation to only the Helium region. The results of a study by Cox, et al. on the instability strip for Cepheids is discussed in LG-2. The strip is expected to be wider than observed since they did not include the effects of convection on pulsation, which limits the red edge of the strip. In their detailed work on the mechanisms for pulsation, they studied the growth of pulsation from noise for a Cepheid model called BK7, from Baker and Kippenhan's earlier work. They initiated the model without any driving, except noise, and it started as a first overtone pulsator. At some later stage, it switched over to the fundamental mode of pulsation at which it stayed in until limiting amplitude. The switch-over may have occurred due to a surge on the computer, at a time sooner than they might have expected, around 350 periods in the fundamental. The maximum kinetic energy in the first harmonic was 3.0×10^{36} ergs as compared to 1.1×10^{43} ergs for the fundamental.

Other developments in nonlinear pulsation codes were pursued in these early days by Alishen (1964) and Hillendahl (1969). Alishen apparently used a sinusoidal inner boundary condition that artificially pumped his models while Hillendahl studied the question of the shocks in the atmosphere of Cepheids but only to a limited extent.

III. FURTHER DEVELOPMENTS IN STELLAR PULSATION CODES

As a continuation of the work of Christy and Cox the author of this review and J. Castor at Cal Tech began working on improved radiative transfer techniques for use in the stellar pulsation codes. Castor's approach was to use the integral method of Schwartzchild while Davis applied the variable Eddington methods of Freemen and Davis (LG-1). Castor was limited in machine-time and only completed one cycle of pulsation with his radiative transfer code (1964(). Bendt and Davis (1971) on the other hand used as a boundary condition the flux and velocity from the diffusion code to study the effects of frequency grouping, zoning, and the weighting of the opacities across the zone interfaces to establish models of Cepheids. The results, in agreement with Castor, showed that the effects of improved radiative transfer in models of Cepheids was minimal. Following this initial work we studied models of RR Lyrae and W Virginis stars using standard boundary conditions, (*i.e.* u(inside)=0.0) instead of results from the diffusion code. The result for S. W. Andromadae was an improvement in U-B vs. phase when compared to the observations. For W Virginis the results were more dramatic (see J Cox's review 1974). Using a model first developed by Christy, Davis noted that shocks formed in the atmosphere of the star resulted in a shoulder on the light curve not present in the diffusion model. This model was also an improvement in that alterations in light maximum and minimum occurred in contrast to only variations in light minimum in Christy's diffusion model. Following the work of Christy and Castor, Hill (1972) studied transfer effects in the extended atmosphere of Christy's RR Lyrae model 5G. He found that a series of shocks formed that could be related to observable lines. The observation of more than one shock forming in the atmosphere of an RR Lyrae model is supported by the earlier work of Hillendahl.

With the ability to do frequency resolved radiative transfer, Cox and Davis looked into the question of the proper averages for the colors from B-V to obtain the temperature of pulsating stars. The results for a model of RR Lyrae supports the observers method of taking <B> - <V> averages to obtain the temperature from the standard relation of Oke, Giver, and

Searle where the opacity was the standard King1A and for Cepheids a new opacity table (CD1) was developed that contained molecules and gave good agreement with the Kraft relation. Other applications of improved radiative transfer techniques were made by Spangenberg, using non-equilibrium diffusion, and Karp, using a modified Henyey method (LG-1).

A study conducted by Vermury and Stothers (1978) and Stothers (LG-4) concluded that Carson opacities and/or the use of tangled magnetic may explain the "bump" sequence using evolutionary masses.

During this development period a method to relax the equations to the periodic solution was devised by Baker and Von Sengbush (1969). The periodic solution of a stellar envelope is calculated using an eigenvalue method. From the resulting Floquet matrix, one obtains the conditions of growth for the various modes of pulsation contained in the model. For an RR Lyrae model Von Sengbush found the blue edge of the instability at limiting amplitude. This method looks very powerful for studies in modal content of pulsation. Stellingwerf (1974) modified the relaxation method to make it more adaptable to the initial value technique. For problems in which e-folding times are long, the method is clearly needed to reduce the number of integrations. Using this method Stellingwerf (LG-1) determined the transition lines and growth rates for a series of RR Lyrae and beat Cepheid models. For a model with $M=0.578M_o$, $L=63L_o$, and $T_{eff}=6500K$, he believed he had established a mixed-mode model, but later attempts by A. Cox et al., using DYNSTAR (LG-4) were unsuccessful. It appears that efforts to find double-mode pulsators using nonlinear methods have proven to be limited to regions in the Hertzsprung-Russell diagram outside those accepted from linear theory for either/or pulsation. Apparently the Stellingwerf code uses an adiabatic spring as an inner boundary condition. Cox (LG-3) found that effects due to this boundary condition appeared in other zones that could have caused problems in the results.

Realizing that standard Lagrangian codes did not resolve the light curves very well a new approach using non-Lagrangian techniques was developed. The original work on dynamic zoning was carried out by Castor but applied by Castor, Davis and Davison (1977) to Cepheid pulsation. Before this work one usually used the calculated velocity profiles for comparison to the bumps observed in the Hertzsprung sequence. The dynamic zoning algorithm improved the light curves considerably. The ability to place zones in the ionization region, which is spatially very thin, results in smoother light curves. Recently, a similar code using temperature instead of mass, as the dependent variable was developed by Aikawa and Simon (1983). Details in the light curves of long period Cepheids have been seen in the results of Moffet and Barnes. For X Cygni, the non-Lagrangian code gives a "dip" as observed on the rising part of the Light curve (0.85 in phase). Interestingly enough the resulting mass is the evolutionary mass ($7.0M_o$) (LG-3).

Methods for treating convection in nonlinear Lagrangian codes have not changed much since the earlier work of Cox et al., (1969). The standard mixing length theory has been modified somewhat by the addition of time dependent terms but still a number of parameters must be fixed to establish the proper effects due to convection. It has been accepted that convection has a small effect near the blue edge of the instability strip but it probably causes the occurrence of the red edge. More recent studies on the effects of convection on the pulsational development of the nonlinear models will be discussed in the next section.

IV. PRESENT STATUS IN NONLINEAR PULSATION

Probably the most exciting part of this review is the discussion of the present status and interest in methods of nonlinear pulsation as applied to problems of "resonance", double-mode pulsation and mass discrepancies. We will consider the various efforts to attack these problems with the use of analytic means, amplitude equations and the use of the very sophisticated relaxation direct integration methods. Probably the most exciting area of research in stellar pulsation at present is the search for the cause of nonlinear development of multimode pulsation as observed in RR Lyrae, beat Cepheids, δ Scuti and possibly other variable stars in the instability strip. Since the early attempts of Stellingwerf (see Section III) many have tried but few have succeeded in producing double-mode pulsation with their nonlinear codes. One exception is the recent model of Buchler and Kovacs with $M=0.85M_o$, $L=35L_o$, and $T_{eff}=6200K$. The period ratio of $P_1/P_0 = 0.756$ does not agree with ratio of 0.746 as observed for this class of double-mode RR Lyrae stars. To obtain this result they had to make a careful selection of the artificial viscosity parameters. We should mention again that attempts to duplicate Stellingwerf's results were unsuccessful (see Section III). Recently, Cox and Ostlie have included a form of time-dependent convection in their code (LG-7) with the expectation that the effects of convection will reduce the amplitude and enhance the occurrence of double-mode pulsators. At present their convective flux differs from Stellingwerf's in an unexplained manner. A good review on various techniques applied to convection is given by Toomrie (LG-5). A new approach to nonlinear pulsation is the use of an asymptotic method by Buchler and Goupil and along with Klapp (LG-7) they have studied the occurrence of resonance in Cepheid pulsation and find good agreement with a set of DYN calculations done by Simon and Davis (LG-5). Davis, Kovacs and Buchler (LG-7) have studied an RR Lyrae model that contains fundamental and first harmonic modes in pulsation. Initiating the model in both fundamental and first harmonic amplitudes (20 km/s) and without "pumping" they observed the detailed growth to limiting amplitude. Using the Maximum Entropy Method (MEM), they analyzed the results at various stages to study the model content of the model. The method provides an accurate practical test for assessing whether a model has relaxed to a steady pulsation and produces time-dependent amplitudes and phases that can be analyzed within the amplitude equation formalism of Goupil and Buchler.

A recent study of interest to this author is that by Aikawa and Simon (1983) where they Fourier analyzed a convection model of an RR Lyrae star, due to Stellingwerf and a radiative transfer model due to Hill (1972). They conclude that the comparisons, for the light and velocity amplitudes, from the convective model agree with their results for a radiative (diffusion) model but the results for the radiative transfer model are in better agreement with observations for RRab variables. They had incorrectly assumed that no other radiative transfer models exited for RR Lyrae (see C. Davis, Variable Star Conference in Budapest - 1975). This result though supports the suggestion by Davis that transfer effects are important in models of RR Lyrae stars.

In these studies on model selection, we need to mention the work of Uji-iye, Aikawa, Ishida, and Takeuti (LG-7). They found that increases in artificial viscosity dissipated the kinetic energy of the first overtone mode much more than the fundamental mode. They were also unsuccessful in constructing a double-mode pulsator. Their Cepheid models had masses from $6.71M_o$ to $3.5M_o$ with constant luminosity, $L=2280L_o$ and $T_{eff}=5850K$.

It appears from these studies that our codes are missing some relevant physics in order to model double-mode pulsators. In this review, many suggestions were made that should be included in one code. First, we should use the direct integration methods coupled with relaxation methods and then add improvements to resolve the shock and ionization fronts (like dynamic zoning) and also improve the radiative transfer prescription as well as the time-dependent convection. With multiprocessing, vectorization, and the new super computers we may be able to assemble a code of this nature in the near future.

Recently, we have also seen a re-birth in an interest to understand the light curves of B. L. Her and W Virginis stars (not to mention δ Scuti and Ap stars). Recent work by Bridger on W Virginis, and Fadeyev and Fokin on B. L. Her and W. Virginis stars (LG-6), have resulted in new insights into the structure of these pulsators. From this reviewers point of view, we believe that improved radiative transfer techniques are needed in order to properly model Pop II class variables (higher luminosity to mass). Radiative transfer is needed in models of these stars to handle the extended atmospheres that results from the stronger shocks that transit during certain phases. The δ Scuti and Ap stars seem to require methods of non-radial pulsation that are not discussed in this review.

Another improvement in our understanding of modal interactions was gained by Whitney and others in the observation that the "echo" description of Christy's for the bump Cepheids is consistent with the resonance ideas of Simon and others.

V. THE FUTURE OF STELLAR PULSATION CODES

With the era of multiprocessing and higher speed computers at hand, the possibilities of many improvements to our nonlinear stellar pulsation codes exists. Improvements in radiative transfer techniques, including the treatment of line transfer, for a more direct comparison to the observations is now possible. The area I believe needs the most improvement and results can already be seen in other applications of astrophysical interest, is in the use of improved hydrodynamics. For one-dimensional Lagrangian codes we can expect improvements in methods of adaptive zoning, mass advection and shock treatments. The recent work of Winkler, Van Leer, Woodward and others is indicative of the improvements possible. The more robust adaptive mesh schemes of Winkler along with improved advection along the lines of MUSCL (Van Leer) or PPM (Woodward) should be incorporated into our nonlinear pulsation codes. The adaptation of Gudonov's method to improve the shock structure is also a possibility. In this new era of development, we can consider thousands of zones or adaptive meshing to resolve the ionization regions and the shocks, as well as improved radiative transfer and the use of tensor viscosities in the treatment of shocks. In the near future, two- and three-dimensional codes will be available with capabilities to do rotation and treat magnetic fields. The time-dependent convection problem may be more tractable in two or three dimensions (Dupree). Methods for treating turbulence are being formulated in two-dimensional codes at present and new techniques such as free Lagrange or Arbitrary Lagrangian Eulerian (ALE), may make the treatment of turbulence and mixing possible.

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